# Jet Control of the Draft Tube Vortex Rope in Francis Turbines at Partial Discharge

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## Abstract

Operating Francis turbines at partial discharge is often hindered by the development of the helical vortex (so-called vortex rope) downstream the runner, in the draft tube cone. The unsteady pressure field induced by the precessing vortex rope may also lead to hydro-acoustic resonance. We introduce in this paper a novel, simple and robust, method to mitigate the vortex rope by using a water jet issued from the crown tip. The jet is supplied with high-pressure water from spiral case inlet, through the tubular shaft. The elimination of severe pressure fluctuations at partial discharge, combined with a significant increase in the draft tube efficiency, compensates the several percents of the overall turbine discharge that bypass the runner. The vortex rope jet control method is investigated using full 3D unsteady numerical simulation, and the benefits of this novel technique are quantified.

## Introduction

The variable demand on the energy market, as well as the limited energy storage capabilities, requires a great flexibility in operating hydraulic turbines. As a result, turbines tend to be operated over an extended range of regimes quite far from the best efficiency point. In particular Francis turbines, which have a fixed-pitch runner, have a high level of residual swirl at the draft tube inlet as a result of the mismatch between the swirl generated by the wicket gates (guide vanes) and the angular momentum extracted by the turbine runner. In the turbine draft tube the flow exiting the runner is decelerated, thereby converting the excess of the kinetic energy into static pressure. The decelerated swirling flow often results in vortex breakdown above a certain level of the swirl number (Ref 1). This vortex breakdown is now recognized as the main cause of the severe pressure fluctuations experienced by hydraulic turbines operating at part load. The pressure fluctuations are caused by the transformation of an axis-symmetrically swirling flow into one or more precessing helical vortices as the operating condition shifts towards part load. The precessing motion of the helical vortex results in a fluctuating pressure on any stationary point of the draft tube cone. In addition, a limited quantity of air or water vapor in the flow provides a degree of elasticity, termed cavitation compliance, and this elasticity can lead to a form of resonance in the draft tube excited by the precessing inhomogeneous pressure field associated with the spiral vortex core (Ref 2). Before introducing our novel method for controlling the vortex rope development at part load and mitigation of the draft tube instabilities, let us review some relevant studies on helical vortex breakdown and practical solutions currently used to address the associated phenomena.

## Vortex rope investigations

The helical vortex breakdown, also known as precessing vortex rope in the engineering literature, benefits from a large body of literature aimed either at elucidating the physics of the phenomenon and building mathematical models, or at developing and testing practical solutions to control the causes and/or the effects. Since in this paper we introduce and evaluate a new technique to control the draft tube flow instability, we will briefly review some relevant studies on both swirling flow hydrodynamics and engineering control solutions.

The self-induced unsteadiness of swirling flow downstream Francis turbine runners at part load has been associated with possible severe flow separation on the blade's suction side. However, the swirling airflow experiments of Cassidy and Falvey (Ref 3) aimed at establishing guidelines for the surge characteristics of hydraulic turbines showed that the spiral vortex breakdown is responsible for the flow unsteadiness. They showed that above a critical swirl number, both the Strouhal number and the pressure amplitude were linearly dependent on the swirl number. A similar radial guide vane apparatus has been used by Nishi et al. (Ref 4) to investigate the water swirling flow in a $9.5^{\circ}$ conical diffuser. The periodic flow field has been measured with a five-hole probe system. They showed that the dimensionless peak-to-peak pressure fluctuation and the corresponding dimensionless fundamental frequency are constant at high cavitation parameter values, but decrease monotonically as vortex cavitation develops. In addition, Nishi et al. (Ref 4) suggest that the circumferentially averaged velocity profiles in the cone could be represented satisfactorily by a model comprising a dead (quasi-stagnant) water region surrounded by the swirling main flow. This model is also supported by the measured averaged pressure, which remains practically constant within the quasi-stagnation region. All these considerations led to the conclusion that the spiral vortex core observed in the draft tube of a Francis turbine at part load is a rolled-up vortex sheet which originates between the central stalled region and the swirling main flow. This vortex breakdown model is further supported by Keller et al. (Ref 5) who showed that the swirling flow with stagnation central core is a solution of the steady axis-symmetric Euler equations for decelerated swirl. Jacob (Ref 6) performed extensive experimental investigations on a Francis turbine model to identify the operating regimes associated with decelerated swirling flow instabilities in the draft tube cone. Kuibin and Okulov (Ref 7) developed an analytical representation for the velocity field induced by a helical vortex in a cylindrical tube. This theory was further developed in (Ref 8), providing a complete mathematical model for confined helical vortices. The most relevant result for flows in draft tube cones is that for a given helical vortex one can determine its precession angular velocity. The main idea is that the helical vortex wrapped on a cylindrical surface induces an axial velocity field co-flowing outside this cylinder and counter-flowing inside the cylinder. The precessing angular velocity therefore adjusts to obtain a quasi-stagnation central core.

The obvious practical importance of predicting the complex flow downstream the turbine runner, in the draft tube, led to the FLINDT research project of Flow Investigation in Draft Tubes (Ref 9). The main objective of this project was to investigate the flow in hydraulic turbine draft tubes, for a better understanding of the complex 3D swirling flow physics and to build up an extensive experimental database describing a wide range of operating points. Full 3D unsteady flow simulations, carefully validated with FLINDT experimental data, led Mauri et al. (Ref 10) to the conclusion that the peculiar sudden drop of the FLINDT draft tube pressure recovery coefficient near the best efficiency operating point could be associated with the Werlé-Legendre separation originating somewhere in the draft tube bend. Susan-Resiga et al. (Ref 11) propose a new analytical representation of the complex swirling flow downstream the FLINDT Francis runner, in very good agreement with experimental data for axial and tangential velocity components. Their model is parameterized only with the discharge coefficient within $\pm 10\%$ the discharge at best efficiency point. It is shown that the swirling flow ingested by the draft tube switches from supercritical to sub critical as the discharge decreases, and the transition matches the discharge where the sudden variation in draft tube recovery coefficient (and consequently in the overall machine efficiency) occurs.

A comparison of experimental and computational results for the FLINDT draft tube at 70% $Q_{BEP}$, where a single vortex rope is fully developed, is presented by (Ref 12). The circumferentially averaged velocity field displays a central quasi stagnation region in the draft tube cone with the vortex rope wrapped around it. The quality of the numerical results is carefully assessed with respect to the vortex rope precession frequency as well as local wall pressure fluctuation amplitudes measured by Arpe (Ref 13). The same operating point of the FLINDT Francis turbine at discharge coefficient $\varphi = 0.26$ and specific energy coefficient $\psi = 1.18$ in cavitation free condition at $\sigma = 1.18$ has been used in our present study, thus being based on a successfully validated numerical model. Note that the unsteady flow simulations in the draft tube required a full 3D coupling of runner and draft tube domains, in comparison with the already classical mixing interface methodology (Ref 14), which couples the steady flows in one inter-blade channel of the turbine distributor with one runner inter-blade channel extended downstream into the draft tube cone. Although expensive, unsteady numerical simulations of draft tube flows at partial discharge, with precessing vortex rope and associated pressure fluctuations, have become a reliable investigation tool over the past few years, which can successfully complement or even replace experimental investigations (Ref 15). Ruprecht et al. (Ref 16), Paik et al. (Ref 17) have proved that present high performance computing capabilities with carefully chosen turbulence models and suitable boundary conditions are able to capture the complex features of 3D precessing helical vortex flows. Sick et al. (Ref 18) report a first successful two-phase flow numerical simulation of cavitating vortex rope, while Iliescu et al. (Ref 19) have shown that two-phase PIV measurements can provide the full 3D unsteady velocity field and allow the accurate reconstruction of the instantaneous cavitating vortex rope geometry together with the determination of the precession frequency.

## Practical solutions for draft tube instabilities

The experimental, theoretical and numerical studies summarized above show that the most important physical aspects of the helical vortex breakdown in a draft tube cone at partial discharge can be quantified and modeled accurately. It is not yet generally agreed on the main causes that lead to self-induced instabilities in swirling flow, therefore the practical solutions that address the draft tube instabilities often have mixed results.

Thicke (Ref 20) reviews some practical solutions for draft tube instability problems. Of course, a proper turbine runner and draft tube design, which brings an associated improvement in turbine efficiency, is essential. A word of caution for machine rehabilitation in an existing power plant, when for economical and safety reasons the spiral casing and draft tube are seldom redesigned: the swirling flow downstream the new runner, further ingested by the old draft tube, may result in a peculiar sudden variation in draft tube pressure recovery coefficient if a swirling flow stability analysis is not carefully performed (Ref 11). Thicke advocates for the runner cone extensions that can be designed to control the pressure and flow below the runner discharge area and thus the swirl in the draft tube. Also, various stabilizer fins attached to the draft tube cone wall are evaluated as beneficial for mitigating the draft tube swirl and pressure fluctuations with some loss in efficiency for low- and medium-head units. To date the fin geometry and configuration is obtained on a trial-and-error basis to minimize the efficiency loss.

Nishi et al. (Ref 21) performed extensive experimental investigations to elucidate the effects of fins on pressure surge by analyzing the wall pressure fluctuations. The results are seen in the framework of a combination of phenomena leading to the draft tube surge, defined as a violent pressure fluctuation having a quasi-synchronous nature caused by resonance: i) the resonance occurring at a critical cavitation parameter when the rotating frequency of the cavitating vortex rope at the elbow section coincides with the natural frequency of the draft tube vibration system; ii) the trigger attributed to the oscillation of the pressure recovery in the foot (downstream diffuser) having the same frequency as the rotating frequency rope. An important conclusion is that the natural frequency cannot always be changed/shifted by the fins. Moreover, severe resonance can actually be introduced by the installation of fins due to the additional cavities volume behind the fins at low cavitation number.

Along the same lines, other solutions propose the introduction of various structures (e.g. splitter plates) in the draft tube cone, aimed at reducing the swirl intensity or destroying the coherent helical vortex. No matter how such structures are designed and tuned, the solution is obviously acceptable for a very narrow range of operating regimes only. Outside this range, the non-adjustable geometrical corrections have adverse effects. It seems that in terms of adjusting the turbine geometry, designing pressure balanced runners with skewed outlets, lately called "X-blade runners" (Ref 22), may provide a certain improvement of the swirling flow stability downstream the runner over an extended operating range. However, this runner improvement provides a reduction in the peak-to-peak amplitude of pressure fluctuations without actually eliminating the phenomenon. While runner design will always be subject for various improvements and compromises, for example in choosing the blade loading distribution from crown to band, the lack of adjustable runner blades leads sooner or later to vortex breakdown at partial discharge.

A widely used surge suppression solution is to inject air in the recirculation region surrounded by the vortex rope, thus producing an essentially axis-symmetric stable flow – a hollow (air) core surrounded by swirling water flow. In the sense of the vortex-breakdown literature, air injection changes the breakdown form from spiral to bubble (Ref 1). Papillon et al. (Ref 23) present practical solutions for natural aeration of hydraulic turbines through the runner cone. This technique seems to have reached industrial maturity by developing reliable heavy-duty spring-free valves, and by performing extensive model tests to assess the influence of air admission on turbine efficiency. It has been proven that relatively small amounts of air have small effects on efficiency while considerably reducing the part-load pressure swings. The air admission effects are often seen in the hydro-acoustic framework (Ref 24). By increasing the volume of the cavity, hence its compliance, the air admission alters the eigenfrequency of the mass oscillations in the draft tube, bringing it away from coincidence with the precessional frequency of the rope. The rope itself, and the excitation mechanism, continues to exist; what is achieved is a "detuning" of the oscillating system thus avoiding the resonance.

An active control technique for pressure pulsations in the draft tube cone has been developed and studied by Blommaert (Ref 25). He injects a small water flow rate (1…2% turbine discharge), modulated by a rotating valve, in the draft tube cone, as a forced excitation intended to cancel the self-induced pressure fluctuations generated by the vortex rope at partial discharge. Once again, the method targets the effects of the pressure fluctuations in the framework of system hydro-acoustics, rather than addressing the main excitation source.

## Jet control of the vortex rope

The above analysis allows us to specify the main requirements for a new technique for controlling the draft tube instabilities specific to Francis turbines operating at part load. Firstly, one should address the main excitation cause, and not the effects. Since the development of the precessing vortex rope as a form of vortex breakdown in decelerated swirling flow downstream the runner is recognized as the excitation source, we should either modify its precessing frequency or mitigate the helical vortex breakdown altogether. Secondly, the control device should not hinder the turbine operation when it is not needed, e.g. at best efficiency operating point or in its neighborhood. As a result, the vortex rope control should be able to switch on/off, or even better to be continuously adjusted, according to the operating regime. Thirdly, in addition to the reduction or even elimination of pressure fluctuations, the control method must not coincide with a significant reduction in the turbine efficiency. Fourthly, the practical implementation should be simple and robust in order to be accepted and implemented in industrial practice with minimal costs and minimal modifications of the current turbine design.

We introduce in this paper a novel flow control technique, which meets the above requirements. *The main idea is to inject a water jet from the tip of the crown cone*, as shown in Fig. 1.

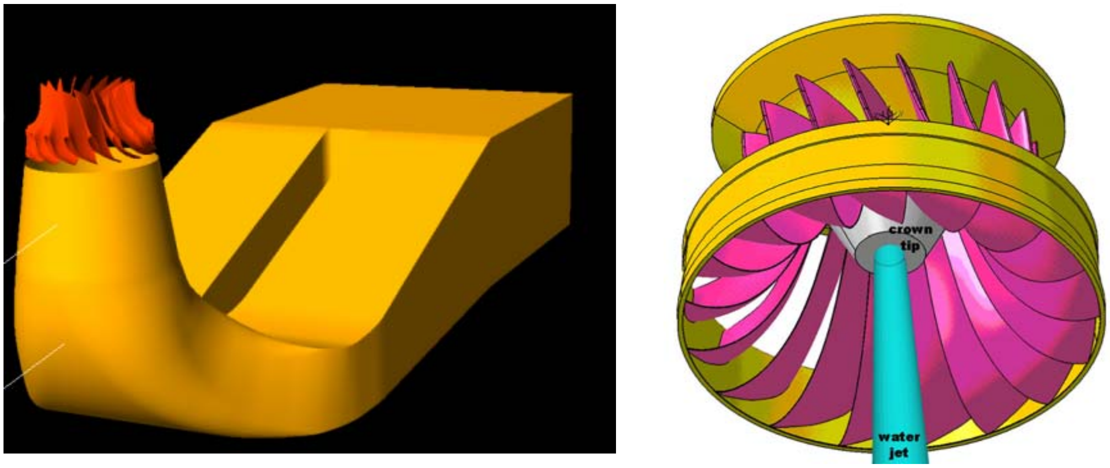
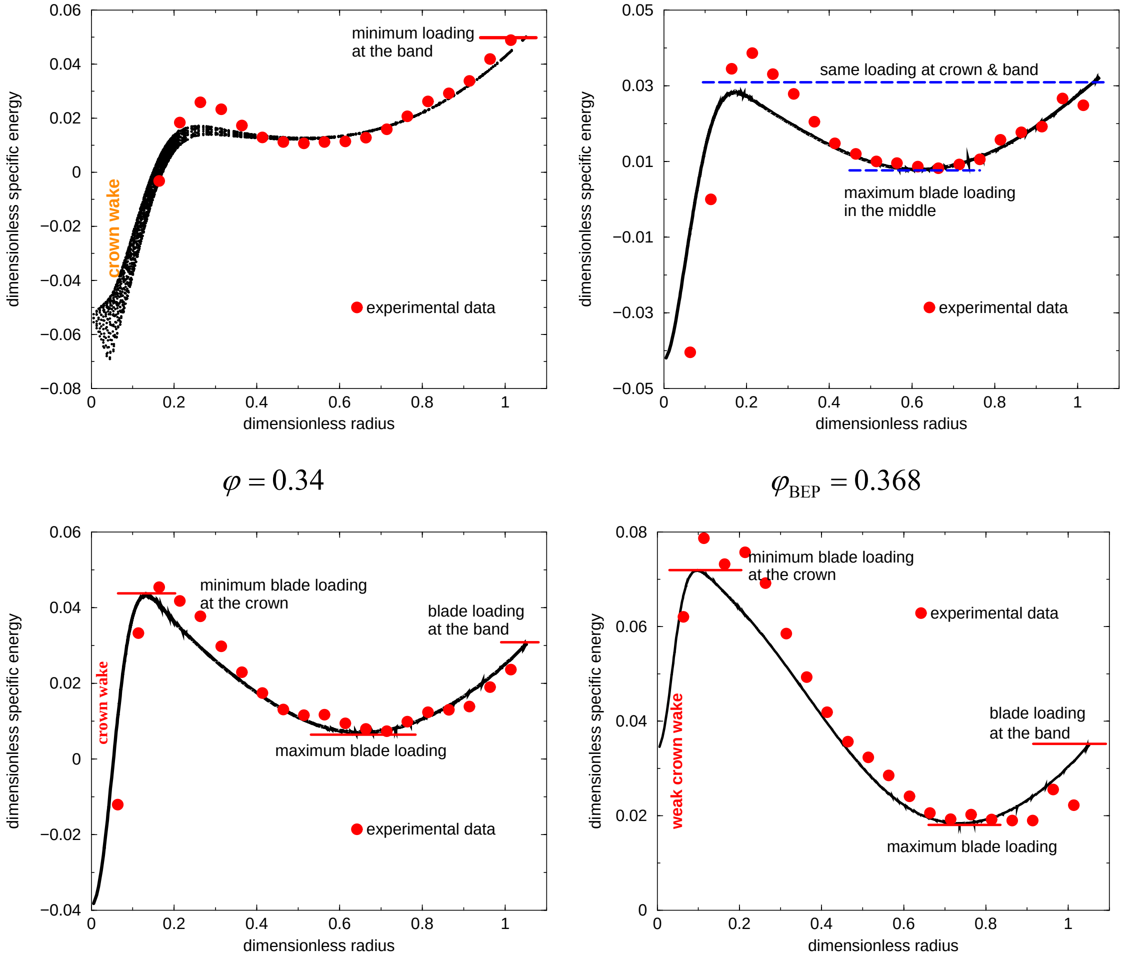


**Figure 1. Computational domain for the draft tube flow analysis, and a detail with the water jet injected at the crown tip.**

Let us first examine qualitatively the jet control technique, according to the requirements stated above. First, the studies on helical vortex breakdown have revealed the development of a central quasi-stagnation region. This phenomenon is associated with the residual swirl downstream of the Francis runner at part load, as a result of a major shift in crown-to-band blade loading, Fig. 2, (Ref 26). While the flow specific energy is approximately constant upstream the runner, at the exit of the runner blades it varies from one streamtube to another. As a result, the specific energy extracted by the runner from the flow has a non-uniform distribution from crown to band. If the specific energy downstream the runner near the band is larger than near the crown, it means that the blade loading is lower at the band than at the crown (e.g. $\varphi = 0.34$), and viceversa (e.g. $\varphi = 0.41$). There is a growing specific energy defect in the crown wake at discharge smaller than $Q_{\mathrm{BEP}}$. Once the helical vortex breakdown occurs, the vortex precession frequency adjusts itself such that the induced axial velocity in the central core is in agreement with the quasi-stagnation region. It results that by injecting a highly energetic jet in the axis one can first change the precession frequency, and eventually by eliminating the quasi-stagnant central region one can avoid the vortex rope development. As a result, the proposed jet control technique addresses directly the vortex rope development thus mitigating the main source of pressure fluctuations, or at least it alters the precession frequency. Injecting a water jet is essentially different from the air admission at the tip of the crown cone since it is aimed at controlling or eliminating the vortex breakdown without necessarily altering the compliance in the hydro acoustic framework. Moreover, the jet accelerates the flow in the crown neighborhood, with the benefit of avoiding (or reducing) the pumping effect near the crown. This "ejector" behavior may actually slightly improve the runner efficiency at part load. Further numerical experiments will assess quantitatively these considerations.



$\varphi = 0.34$ $\varphi_{\mathrm{BEP}} = 0.368$

$\varphi = 0.38$ $\varphi = 0.41$

**Figure 2 Radial distribution of the dimensionless specific energy $2\left(p/\rho + V^2/2\right)/\left(\omega D/2\right)^2$ in a survey section downstream a Francis turbine runner.**

Secondly, when it is not needed, the water jet can be switched off. The jet control technique does not require geometrical modifications of the runner, and no other devices need to be installed in the draft tube cone. The solution we are proposing is based exclusively on hydrodynamic swirling flow control.

Thirdly, by avoiding the helical vortex breakdown the overall performance of the draft tube at part load is significantly improved by reducing the hydraulic losses due to severe flow non-uniformities and unsteadiness. For example, according to the hill chart of the Francis turbine investigated in this paper, (Ref 12, Fig.2), the global efficiency at 70% part load is slightly smaller than 83%, or more than 10% lower than the BEP efficiency. Moreover, the draft tube pressure recovery coefficient, (Ref 11, Fig.3) drops from 0.7 at $\varphi_{\mathrm{BEP}} = 0.368$ to approx. 0.1 at $0.7 \times \varphi_{\mathrm{BEP}} = 0.26$. Clearly there is room for improvement when using the jet control technique.

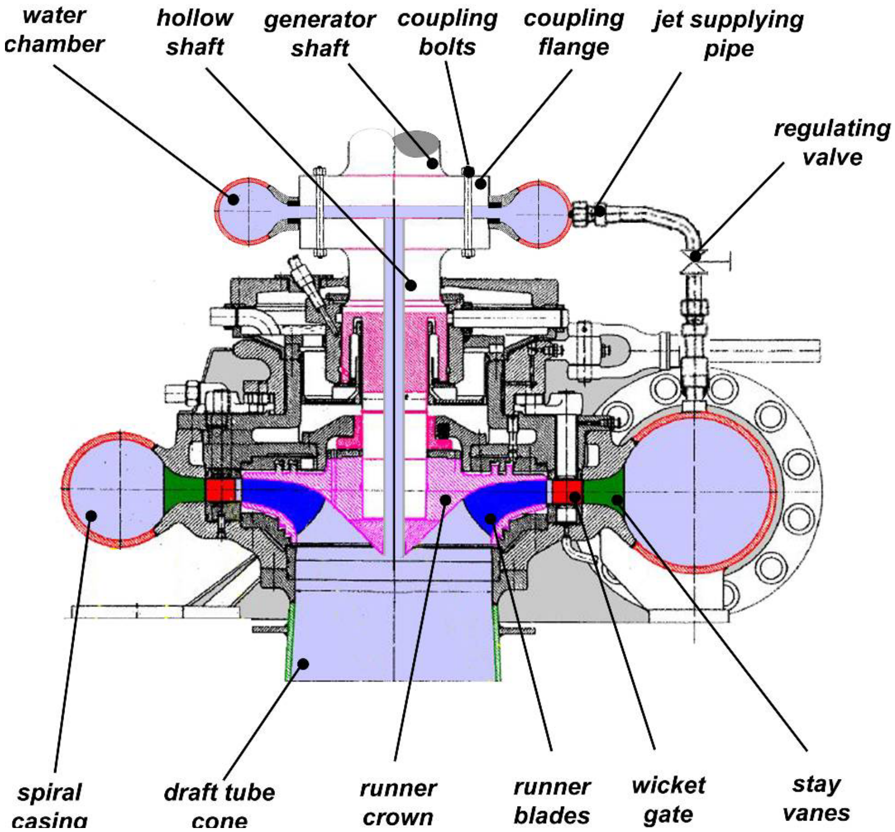


**Figure 3. Technical solution for jet control of the flow in draft tube cone.**

Fourthly, the technical solution for producing the water jet at the crown tip takes advantage of the hollow turbine shaft, and benefits from a high-pressure water supply from upstream the turbine spiral casing. The water is introduced in the shaft using a water chamber installed on the coupling between the turbine and generator shafts. In addition, a regulating valve adjusts the jet flow rate according to the current turbine operating regime. This technical solution is sketched in Fig. 3. It is important to stress once again that the jet control technology does not alter the geometry of the turbine water passage, and does not use additional structural elements within the draft tube. When the jet control system is turned off by the regulating valve, the turbine operates as usual.

The jet control technique proposed here provides an active control of the swirling flow downstream of the runner. It actually uses a fraction of the overall turbine discharge. The jet discharge actually bypasses the turbine bladed region and produces no power at the turbine shaft. However, optimization of jet parameters can actually provide a turbine efficiency increase that compensates the hydraulic energy spent on the jet. In addition, of course, we have the benefit of reducing or eliminating the severe pressure fluctuations at partial discharge, with a significant increase in the turbine safety.

### Considerations on the jet parameters

For the jet control method proposed and analyzed in this paper, the jet issued at the crown tip, Fig. 1, is supplied with water from upstream the spiral case. As a result, the maximum jet velocity can be roughly estimated as

$$V_{\text{jet}} \approx \sqrt{2gH} = \omega R\sqrt{\psi}\ , \tag{1}$$

where $\psi$ is the energy coefficient, $R$ and $\omega$ are the runner outlet radius and the angular speed, respectively. Losses in the flow passage supplying the jet as well as the actual static pressure generated by the turbine at the hub both influence the actual jet flow velocity. A jet flowrate control mechanism will be necessary to obtain the optimal flow for every operating condition.

## Numerical analysis of the jet control technique

As mentioned above, we used the FLINDT runner-draft tube combination for the Computational Fluid Dynamics calculations (CFD). The configuration for the calculation contained the full runner and the draft tube meshes, Fig. 1. This was considered to be the minimum extent of the flow domain that needed to be included for a prediction of the draft tube rope. The commercial CFD code CFX10 was used with the "High-resolution" differencing scheme in space, the second order backward Euler transient scheme and $k$-$\varepsilon$ turbulence model. This numerical approach has been validated in (Ref 12), the only change being the change of the solver from CFX5.6 to CFX10. Hence, more details on the numerical approach as well as its level of validity can be found in that paper. All calculations performed where single phase, i.e. assuming that no vapor is present in the flow.

Figure 4 shows in a qualitative way what the influence of the axial hub jet is. On the top left we see a nicely developed single helical draft tube vortex rope. On the top right, where the jet is in operation, the central low pressure region indicated by the iso-surface has been greatly reduced and its shape has changed from clearly helical to a slightly off-centre extended cone. The same effect is visible in the two images at the bottom of Figure 4 in the form of a pressure distribution on the draft tube elevation plan. The precessing low-pressure region remains much closer to the draft tube cone axis once the jet is in operation. As a result wall pressure fluctuations have to be lower than without the jet.

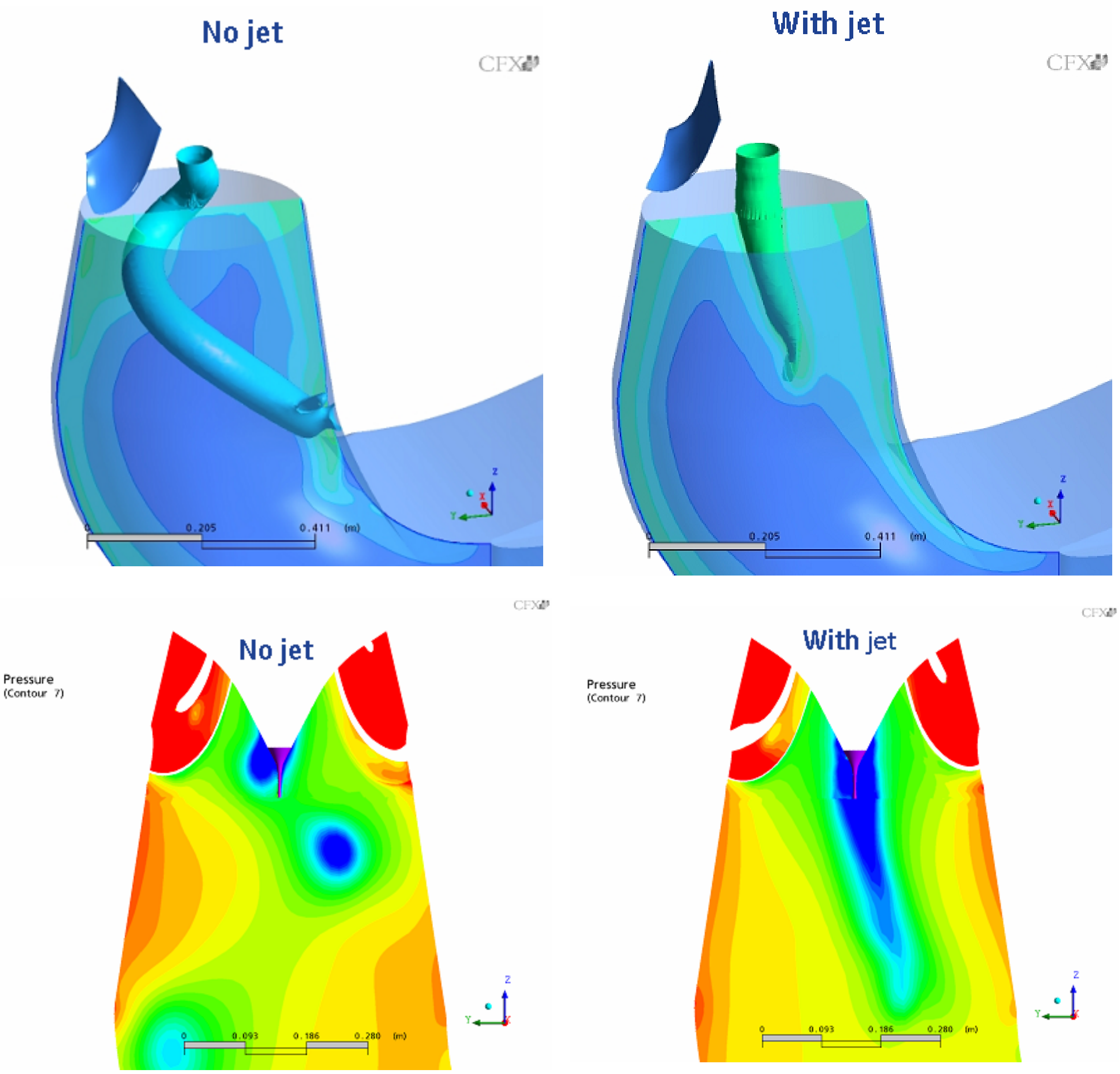


**Figure 4. Draft tube rope at part load condition without (left) and with (right) axial jet (top: 3D instantaneous iso-pressure surface and velocity distribution, bottom: instantaneous pressure distribution)**

In the time signal depicted on the left of Figure 5 we can clearly see the reduction of the draft tube cone wall pressure fluctuation once the jet is in operation. This time signal shows the evolution of the static pressure at a specific location on the draft tube cone from the start of the unsteady CFD calculation to the operation of the jet. It takes a large number of time steps before a well-established periodic solution is reached. After the beginning of the jet flow, both the amplitude and the frequency of the pressure fluctuation have changed. The image on the right in Figure 5 depicts a time signal of the torque of an individual blade. Here we can also see a significant reduction in amplitude.

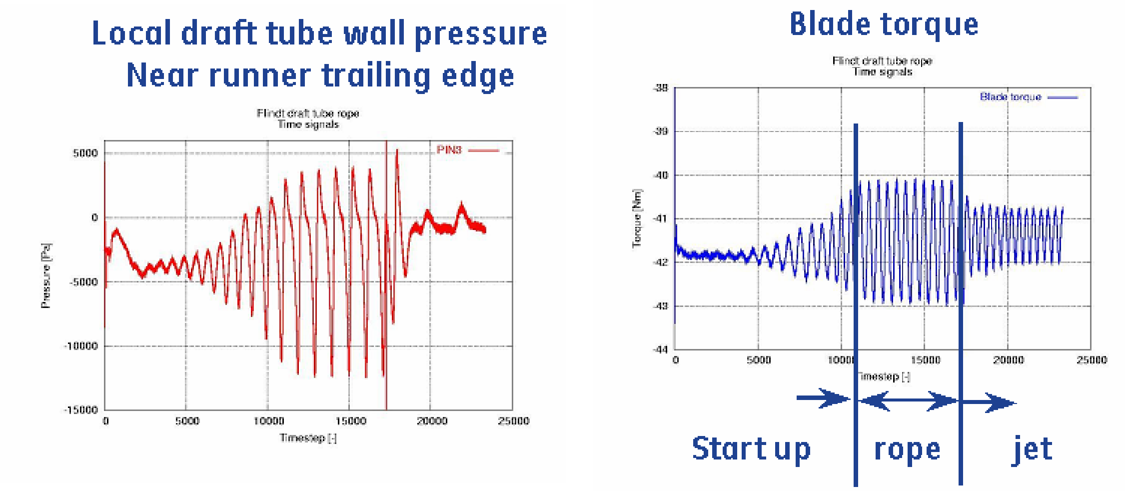


**Figure 5. Time signal evolution in CFD calculation from start up over developed periodic rope to operation with jet**

Figure 6 shows how the axial jet changes the mean velocity profiles. In our study we tested a number of different jet diameters and jet flow rates, ranging 6-28% $D_{jet}/D_{throat}$ for the diameter and 1-28% $Q_{jet}/Q$ for the flow rate. Within this range we were able to find a combination that is technically reasonable while achieving the desired effect of significantly reducing the pressure fluctuations caused by the draft tube rope. The flow that is used for the jet is lost for power generation. Consequently the effect of the jet on the energy balance is of interest. From the jet flow rate range given above it is clear that a jet that sufficiently reduces the pressure fluctuations corresponds to large efficiency loss. However, the jet at part load improves both the draft tube and the runner flow, such that overall efficiency losses due to jet operation were only in the order of 0.2%.

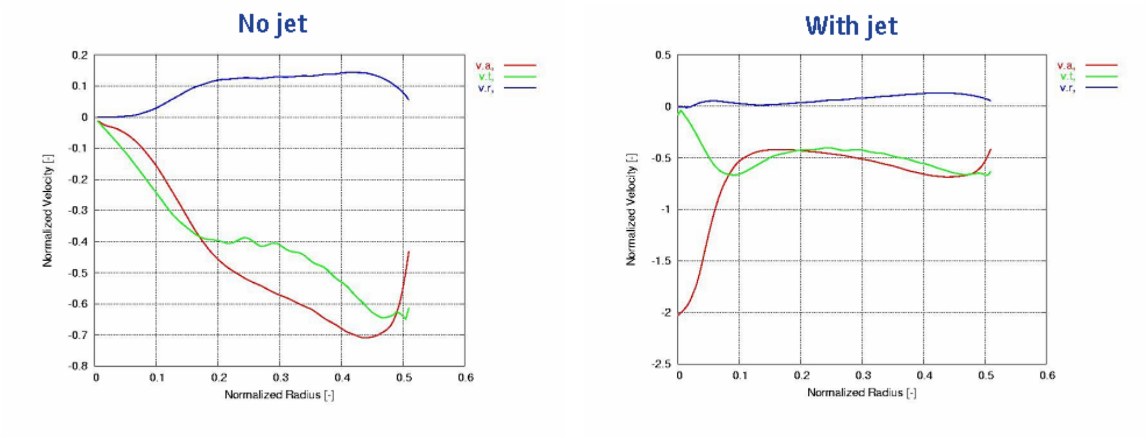


**Figure 6. Velocity profiles at draft tube inlet**

## Conclusions

The paper introduces a novel method for mitigating the severe flow field fluctuations in Francis turbines operated at part load. The analysis of theoretical and experimental studies on decelerated swirling flows in the turbine draft tube cone, led us to the conclusion that the occurrence of the helical vortex breakdown at part load is directly related to the severe flow deceleration at the axis downstream of the runner. As a result, we propose to inject a water jet from the tip of the crown cone in order to mitigate the draft tube instability. In comparison with other solutions which address the same problem, the jet control of the flow is shown to eliminate the well known drawbacks while presenting the following main advantages: a) it successfully addresses directly the main cause of the flow instability, rather than the effects; b) it does not require geometrical modifications of the runner, and no other devices need to be installed in the draft tube; c) it is continuously adjustable according to the operating point, and it can be switched-off when it is not needed; d) the practical implementation is simple and robust; e) although a fraction of the discharge bypasses the bladed region, the overall turbine efficiency does not suffer thanks to the improvement in both runner and draft tube efficiencies when the jet is on. The effectiveness of our jet flow control in draft tubes is quantitatively evaluated through reliable and accurate full 3D unsteady flow numerical simulations, showing at least one order of magnitude reduction in wall pressure fluctuations at part load, while the overall turbine efficiency is practically unchanged.

### Acknowledgement

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